original memoirs. The elasticity of pure graphite was determined specially by the author, and the velocity of sound where not known has been calculated from E and ρ. The conductivities of crystals though functions of elasticity are too complex to satisfy the above relations.

Material.	Density, ρ.	Elasticity, $E \cdot 10^{-11}$.	Velocity of Sound, V. cm.sec.10^{-5}.	$E\rho \times 10^{-11}$.	$V^2\rho^2 \times 10^{-11}$.	Thermal Conductivity in calories Observed
Quartz	2·66, K.L.	6·8, K.L.	5·0, cal.	18·1	17·7	{ 23.10 ³ ᵇ. 16 ⊥, K.L.
Flint Glass	2·9, K.L.	4·8, L.B.	4·1, K.L.	14·1	14·1	14·3, L.B.
Crown ,, ...	2·5, K.L.	7·3, L.B.	5·4, K.L.	18·3	18·3	18·3, L.B.
Soda ,, ...	2·8, K.L.	7·9, L.B.	5·3, cal.	22·2	22·0	22·2, L.B.
Graphite	2·3, T.	5·25 T.	4·65, cal.	12·1	11·5	12·0, K.L.
Marble	2·7, K.L.	2·6, K.L.	3·10, cal.	7·0	7·0	7·1, K.L.
Mahogany, Hond.	·62, K.	·88, K.L.	3·7, cal.	0·54	0·52	0·5, K.L.
Deal	·5, K.L.	·9, K.L.	4·2, L.B.	0·45	0·44	0·4, K.L.
Ice	·91, K.L.	·28, K.L.	1·74, cal.	0·25	0·25	0·22. E.
Paraffin Wax ...	·91, L.B.	·151$_{17}$,L.B.	1·30, L.B.	0·137	0·14	0·141, L.B.
V. Indiarubber...	1·02, K.L.	·05, K.L.	·07, K.L. ·7, cal.	0·05	0·0005 0·05	0·45, K.L.
Cork	0·24, K.L.	·0055, cal.	·48, L.B.	0·0013	0·0013	0·13, K.L.

It was not to be expected that a quasi-solid such as indiarubber should conform to the rule. Its velocity of sound calculated from E and ρ is $0·7 \cdot 10^5$ centimetres a second; the observed value (K.L.) is $0·07 \cdot 10^5$. In either case the values of $E\rho$ or $V^2\rho^2$ are of a lower order than the conductivity observed by Lees, though the numeral is the same.

The elasticity of cork calculated from the velocity of sound is $0·0055 \cdot 10^{11}$. Its value for steady stress is about one half of this; $E\rho = ·00132 \cdot 10^{11}$, or exactly one hundredth of Lees' value 0·13. The cellular structure of cork makes it difficult to say how far the vibrations are transmitted by the walls or contained air. Baking the cork to a dark brown softens it and improves its value as a heat insulator. The cell-walls are then seen under the microscope to be slack and collapsed, so that they undoubtedly have some part in the transmission. On the other hand, pumice-stone has a conductivity much the same as cork and has a more rigid structure.

The agreement between the values of k, $E\rho$, and $V^2\rho^2$, for substances of such different composition as quartz and wax, indicates that there is some fundamental process, a consequence possibly of the electrical structure of matter, certainly independent of molecular structure, beneath these phenomena. On the electron theory of matter mechanical forces are

eventually expressed in terms of electric charge or the movement of charge. Resistance to compression is a repulsion between the structural elements of matter possibly of the nature of an electrostatic repulsion between electrons which form the outer frame of atoms. The sudden compression of a sound-wave then gives rise to electrostatic forces of recoil and the velocity of sound is the rate of uniform transmission of these through a loaded æther. On the other hand, the heat energy of irregular motion may be considered as that of alternating atomic currents attracting and repelling at random and so transmitting the motion electromagnetically.

It is possible to extend this to a general consideration of thermal conductivity in insulators as an electromagnetic phenomenon. The object of the present note is to direct attention to the above relations between k, E, ρ, and V which must be explained by any theory of conduction in non-metallic solids. The chief practical qualification of a material as a heat insulator is that it should be light and inelastic.

LXXIV. *A Positive Ray Spectrograph.* By F. W. Aston, M.A., D.Sc., Clerk Maxwell Student of the University of Cambridge[*].

[Plate IX.]

THE analysis of positive rays by electric and magnetic fields giving deflexions at right angles to each other has been very completely worked out by Prof. Sir J. J. Thomson[†]. Alternative methods have been suggested by Dempster[‡] and others. Dempster's arrangement depends on the knowledge of the potential through which the rays have fallen in the discharge-tube, and is therefore only practicable in the case of low velocity rays.

Positive rays obtained from an ordinary discharge-bulb vary both in mass and velocity. An electric field will spread them into an 'electric spectrum' with deflexions proportional to $\dfrac{e}{mv^2}$; a magnetic field will spread them into a 'magnetic spectrum' with deflexions proportional to $\dfrac{e}{mv}$. In Thomson's method of crossed deflexions, in which both fields are applied simultaneously, rays having constant $\dfrac{e}{m}$ will lie on parabolas[§]

so that masses can be compared by measuring the ordinates. This method, though almost ideal for a general survey of masses and velocities, has objections as a method of precision, many rays are lost by collision in the narrow canal-ray tube, the mean pressure in which must be at least half that in the discharge-bulb; very fine tubes silt np* by disintegration under bombardment; the total energy available for photography falls off as the fourth power of the diameter of the canal-ray tube.

The first two can be overcome, as will be described below, by replacing the brass or copper tube by fine apertures made in aluminium, a metal which appears to suffer no appreciable disintegration, and by exhausting the space between these apertures to the highest degree by means of a subsidiary charcoal tube or pump. The falling off in intensity of the parabolas as one attempts to make them finer is a very serious difficulty, as the accuracy and resolving power depend on the ratio of the thickness to the total magnetic deflexion; and if we increase the latter the electric deflexion must be increased to correspond and the parabolas are drawn out, resulting again in loss of intensity.

Methods of increasing the intensity of the spot.

The concentration of the stream of positive rays down the axis of the discharge-bulb is very marked, but there is good evidence for assuming that the intense part of the stream occupies a fairly considerable solid angle. This suggests the possibility of an increase of intensity by means of a device which should select the rays aimed at a particular spot on the plate whatever direction they come from. For example, a thin gap between two coaxial equiangular cones would allow the rays to be concentrated at the vertex. The dimensions of the patch formed would be roughly those of one given by a cylindrical canal-ray tube of diameter equal to the width of the gap. The increase of intensity would therefore be considerable; but the method is not easy to put into practice, and, in the case of deflexions through large angles, would necessitate a curved photographic surface.

Clearly the simplest way of increasing the intensity of the spot without increasing its dimensions, at any rate in one direction, is to use two parallel straight slits. In the case of the method of crossed deflexions this device would only be of use in a special case such as the resolution of a close double, as the parabolas will only be sharp at points where they are parallel to the slit.

* *L. c.* p. 21

Possibilities of focussing positive rays.

The very great accuracy attained in the spectrometry of light depends largely on the fact that a considerable solid angle of divergent rays from a point source can be brought to a point image by means of a lens. It is of importance to inquire if any such convergence can be applied to rays of charged particles by any electric or magnetic device.

As regards the ordinary lens the problem appears rather hopeless, but electric or magnetic analogues of the cylindrical lens can be made theoretically in several ways.

Thus magnetically homogeneous rays diverging from a point or slit source S (fig. 1) will be brought to a first-order

Fig. 1.
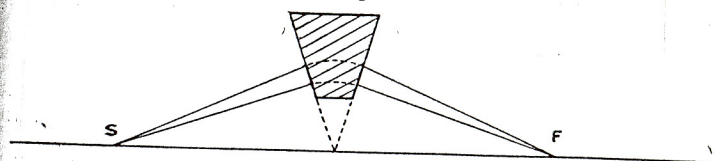
focus F if the integrated intensity of the field traversed by any one ray is proportional to its angular distance from the line SF, *e. g.* by the use of parallel pole-pieces of wedge-shaped section.

Such a 'magnetic lens' is not of much immediate value as the magnetic spectrum of positive rays is very complex. The electric spectrum, on the other hand, possesses one very important simplicity, namely, that the distribution of intensity in it is a property of the discharge rather than of the particles carrying it, and is to a great extent the same for all particles: in other words, the value of mv^2 giving the brightest result for particles of one mass will in general give the brightest result for all.

Fig. 2.
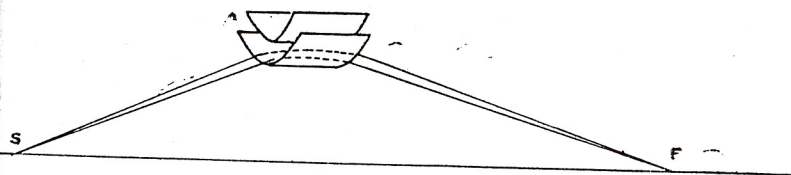
A variety of electrical analogues of the cylindrical lens are possible. In fig. 2 divergent rays of constant mv^2 passing through the field between two charged plates whose sections are two concentric coaxial rectangular hyperbolas will be brought to a focus on the line passing through the source

and the centre of the hyperbolic system. For the field
between plates of this form is such that its intensity varies
directly as the distance from this line.

Again in fig. 3 a beam of such rays generated by the use

Fig. 3.
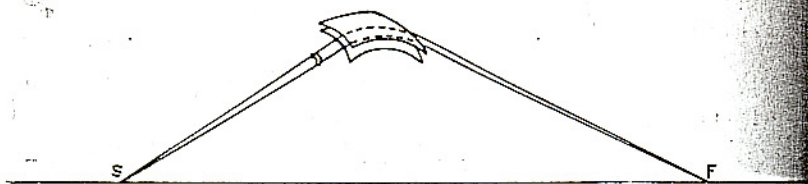
of a point source and an annular slit, if passed between two
charged concentric spherical surfaces, will travel in great
circles and come to a focus on the axis of the system.

Principle of the Spectrograph.

The above devices will give bright spectra of the rays in
terms either of $\frac{mv}{e}$ or $\frac{mv^2}{e}$; what is actually required is an
arrangement which will give a spectrum in terms of $\frac{m}{e}$ only
and, at least over a small range, independent of v. This is
done in the present apparatus as diagrammatically indicated

Fig. 4.
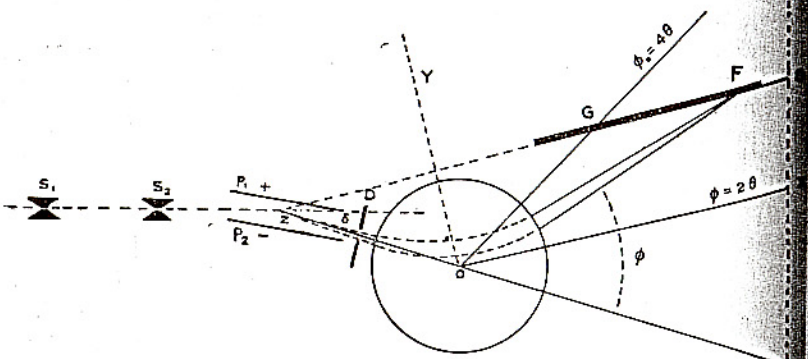
in fig. 4. The rays after arriving at the cathode face pass
through two very narrow parallel slits of special construction
$S_1 S_2$, and the resulting thin ribbon is spread out into an

electric spectrum by means of the parallel plates P_1, P_2.
After emerging from the electric field the rays may be taken,
to a first order of approximation, as radiating from a virtual
source Z halfway through the field on the line $S_1 S_2$. A group
of these rays is now selected by means of the stop or dia-
phragm D, and allowed to pass between the parallel poles of
a magnet. For simplicity the poles are taken as circular,
the field between them uniform and of such sign as to bend
the rays in the opposite direction to the foregoing electric
field.

If θ and ϕ be the angles (taken algebraically) through
which the selected beam of rays is bent by passing through
fields of strength X and H, then

$$\theta v^2 = l X \frac{e}{m} \quad (1), \quad \text{and} \quad \phi v = L H \frac{e}{m} \quad (2),$$

where l, L are the lengths of the paths of the rays in the
fields. Equation (1) is only true for small angles, but exact
enough for practice. It follows that over the small range of
θ selected by the diaphragm θv^2 and ϕv are constant for all
rays of given e/m, therefore

$$\frac{\delta\theta}{\theta} + \frac{2\delta v}{v} = 0, \quad \text{and} \quad \frac{\delta\phi}{\phi} + \frac{\delta v}{v} = 0,$$

so that

$$\frac{\delta\theta}{\theta} = \frac{2\delta\phi}{\phi},$$

when the velocity varies in a group of rays of given e/m.

In order to illustrate in the simplest possible way how this
relation may be used to obtain focussing, let us suppose the
angles (exaggerated in the diagram) small and the magnetic
field acting as if concentrated at the centre O of the pole-
pieces. If the length $ZO = b$, the group selected will be
spread out to a breadth $b\delta\theta$ at O, and at a further distance r
the breadth will be

$$b\delta\theta + r(\delta\theta + \delta\phi) \quad \text{or} \quad \delta\theta\left[b + r\left(1 + \frac{\phi}{2\theta}\right)\right] \quad . \quad . \quad (3)$$

Now as the electric and magnetic deflexions are in opposite
directions, θ is a negative angle. Say $\theta = -\theta'$. Then if
$\phi > 2\theta'$, the quantity (3) will vanish at a value of r given by

$$r(\phi - 2\theta') = b \cdot 2\theta',$$

which equation appears correct within practical limits for
large circular pole-pieces.

Referred to axes OX, OY the focus is at $r \cos(\phi - 2\theta')$, $r \sin(\phi - 2\theta')$, or $r, b . 2\theta'$; so that to a first-order approximation, whatever the fields, so long as the position of the diaphragm is fixed, the foci will all lie on the straight line ZF drawn through Z parallel to OX. For purposes of construction G the image of Z in OY is a convenient reference point, ϕ being here equal to $4\theta'$. It is clear that a photographic plate, indicated by the thick line, will be in fair focus for values of e/m over a range large enough for accurate comparison of masses.

The arrangement, which has a distinct resemblance to the ordinary quartz spectrograph, gives very complete control. The field between the plates can be adjusted to allow the brightest part of the electric spectrum to be used which, as has been shown, is in general the same for all normal rays under steady discharge, and the values of e/m can be compared very accurately from the positions of their lines relative to those of standard elements which can be brought to any desired position on the plate by varying the magnetic field strength.

Preliminary results.

In order to test the method before making the somewhat elaborate camera, a temporary apparatus was set up using an existing camera, the plate being in the position indicated by the dotted line AB. Under these conditions the focus can only be good at or near the point B. The results so far obtained are exceedingly promising, and show that as far as intensity and sharpness of the lines are concerned no serious difficulty need be apprehended. Plate IX. fig. 1 is a photograph taken with an electric field but with no current passing through the magnet. It shows the undeflected spot as a sharp bright line—with a patch of fog above it due to some internally reflected light—and the electric spectrum of the positive rays spread out below. In fig. 2 the magnet has been turned on, other conditions being identical with fig. 1. It will be seen that although there is no diaphragm in this apparatus and therefore practically the whole of the electric spectrum is in use, yet the rays corresponding to the hydrogen molecule are concentrated as a sharp bright line 1·4 cm. above the undeflected spot and displaced a little to the right as the magnetic pole-pieces are not set truly vertical. Fig. 3 was taken with a smaller field, showing the hydrogen atom near the focal point and the molecule below it very much out of focus.

Plate IX. fig. 4 was taken with a much larger magnetic field, the bright line with its four fainter companions is due to carbon and its compounds CH, CH_2, CH_3, CH_4. Lines corresponding to still heavier particles are seen as indistinct patches which would come up in turn to the focal point if the magnetic field were still further increased.

These results were obtained with residual gas from charcoal, the slits were ·05 mm. wide, and the current in the main discharge-tube roughly of the order of one milliampere at 30,000 volts. The duration of exposure was 2 minutes in the first three cases and 8 minutes at a rather lower current in the fourth.

These remarkably short exposures indicate clearly the great advantage obtained by the combined use of a slit system and a focussing arrangement.

The other parts necessary to complete an apparatus suitable for a general investigation of the relative masses of positive rays are now being constructed, with which it is hoped to obtain results comparable in accuracy with those determined chemically. For masses not too great or widely separated an accuracy of one tenth per cent. is by no means impossible. If anything like this order is obtained, the composition of atmospheric Neon—element or isotopic mixture—will be settled beyond dispute (one of the prime reasons for this work) and several other problems laid open to direct attack.

It is as well to point out in view of future developments, that second-order corrections in focus are clearly possible by varying the section or even the figure of the pole-pieces and electric plates; but it is not proposed to employ these refinements until such incidental difficulties as small stray fields due to electrification by the rays themselves have been successfully overcome.

In conclusion the author wishes to acknowledge his indebtedness to many friends for their kind advice and help, in particular as regards the mathematical analysis of some of the more complex systems considered which led up to the one finally adopted. Also to the Government Grant Committee of the Royal Society for defraying the cost of parts of the apparatus used in the practical investigation.

Summary.

Precision methods of positive ray analysis are discussed and means of improving the brightness of the beam analysed are suggested.

Theoretical electrical and magnetic analogues of optical cylindrical lenses for focussing positive rays are shown to be possible.

A form of positive ray spectrograph giving a focussed spectrum depending solely on ratio of mass to charge is described.

Actual photographic results obtained with a preliminary apparatus are submitted showing the great accuracy possible by the method with which it is hoped to compare masses to one tenth per cent.

Cavendish Laboratory.
August 1919.

Appendix.

The Construction of the Slit System.

The very fine slits used in this apparatus were made with comparative ease as follows:—A cylinder of pure aluminium about 10 mm. long by 5 mm. wide is carefully bored with a hole 1 mm. diameter. The resulting thick-walled tube is then cleaned and crushed with a hammer on an anvil until the circular hole becomes a slit about ·3 mm. wide. Continuation of this treatment would result in a slit as fine as required giving the maximum resistance to the passage of gas, but its great depth would make the lining up of a pair a matter of extreme difficulty. The crushed tube is therefore now placed between two V-shaped pieces of steel and further crushed between the points of the V's at about its middle point until the required fineness is attained. Practice shows that the best way of doing this is to crush until the walls just touch, and then to open the slit to the required width by judicious tapping at right angles to that previously employed. With a little care it is possible to make slits with beautifully parallel sides to almost any degree of fineness, ·01 mm. being easily attainable. At this stage the irregularly shaped piece of aluminium is not suited to accurate gas-tight fitting; it is therefore filled with hard paraffin to protect it from small particles of metal &c., which if entering cannot be dislodged owing to its shape, and turned up taper to fit the standard mountings. These in the present apparatus are taper-holes in the back of the cathode and in a corresponding brass plug at the ends of a wide tube 10 cm. long. When turned, the paraffin is easily removed by heat and solvents.

LXXV. *On Some Aspects of the Theory of Probability.* By DOROTHY WRINCH, *Lecturer at University College, London,* and HAROLD JEFFREYS, *M.A., D.Sc., Fellow of St. John's College, Cambridge* *.

I. *The Nature of Probability.*

THE theory of probability suffers at the present time from the existence of several different points of view, whose relations to one another have apparently never been adequately discussed. On the one hand some authorities follow de Morgan and Jevons in regarding probability as a concept comprehensible without any definition, and perhaps indefinable, satisfying certain definite laws the logical basis of which is not yet clear. On the other hand, attempts have been made to give definitions of probability in terms of frequency of occurrence; of these one is due to Laplace, who was largely followed by Boole, and another to Venn. Frequency of occurrence being a well-understood mathematical concept, such a definition would be important if it could be carried out; for then the undefined notion of probability would be expressed in terms of others that are better understood, and its laws, if true, would become demonstrable theorems in pure mathematics instead of postulates. Thus the subject would acquire the certainty of any other portion of pure mathematics and it would be unnecessary to investigate its foundations independently. It appears, however, as we hope to show in the first part of the present paper, that the definitions offered either implicitly involve the very notion they are meant to avoid, or else make assumptions which are actually erroneous. We therefore consider it best to regard probability as a primitive notion not requiring definition.

Laplace defines probability † as the ratio of the number of favourable cases to that of all possible cases, and then goes on to say "but that supposes the various cases equally possible," so that to understand this definition it is necessary to examine what Laplace meant by *equally possible*. The expression is meaningless as it stands, for a proposition relative to a set of data is always either possible or impossible; there can be no degrees of possibility. He indicates later that if a coin is unsymmetrical the probability of throwing a head may be greater than that of throwing a tail, though the difference may be small; yet both are

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* Communicated by the Authors.

† *Théorie analytique des probabilités*, troisième édition, p. 7 of introduction.

terraces of Barbados were examined chemically and microscopically, in order to ascertain the composition, nature, and origin of their extraneous mineral contents. A special method was used, whereby the extraneous mineral matters were separated, practically without alteration, from large quantities of the limestones. Chemical analyses of the residua were made, and the results of these and of the microscopical examinations are tabulated in the paper. The extraneous minerals present were found to be apparently-fresh and largely-unaltered fragments of wind-borne volcanic minerals and glass. It was found that the volcanic minerals enclosed in the reef-corals on which they fell have been protected from change; those in the clastic limestone or bed-rock show signs of detrition and weathering prior to the consolidation of the limestone. Similar minerals separated from clay normally formed and accumulated in a pothole in the limestone supply evidence of weathering changes after being set free from the rock. It is shown that the composition of the sedentary residual soils on the higher limestone-terraces of Barbados corresponds in its essential parts with the residua separated, either naturally or artificially, from the limestone.

The proportions of magnesium carbonate present in the coral-rock are briefly discussed, and complete analyses of the high-level and the low-level limestones are given. A note on the proportions of titanium oxide in the Barbados Oceanic clays and in some of the *Challenger* and *Buccaneer* deep-sea dredgings is appended to the paper.

LXXX. *Intelligence and Miscellaneous Articles.*

ON A NEW FORM OF CATENARY.

To the Editors of the Philosophical Magazine.

DEAR SIRS,—

MAY I be permitted through you to express to Mr. McLeod my sincere apologies for having unintentionally published my paper "On a New Form of Catenary" when the subject had already been dealt with by him.

I was requested to undertake the investigation during the summer of 1917, and a summary of it was incorporated in a monthly report of the R.N.A.S. Establishment, Cranwell, early in 1918.

This was a highly confidential document and was never published; and since then the paper has been lying by awaiting permission for open publication, which was only granted this summer.

I may add that I was quite unaware of Mr. McLeod's work until I read his letter in your November issue.

Yours faithfully,
J. HOLLINGWORTH.

College of Technology,
Manchester,
Nov. 10th, 1919.

FIG. 1.

FIG. 2.

FIG. 3.

FIG. 4.