Energy Levels of a Vector Meson in a Smeared Coulomb Field

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CORBEN and Schwinger[1] have derived the radial equations for a vector meson in a central field. They show that the states of a vector meson having a definite j (total angular momentum) and a definite parity fall into two groups. One is a group for which $l=j$ (l is the orbital angular momentum). These states give the ordinary scalar meson levels. The second group of states contains linear combinations of states for which $l=j-1$ and $l=j+1$. Corben and Schwinger have shown that solutions for the latter group of states, in a pure Coulomb field, exist only for the special case $j=0$.

Since, in reality, the Coulomb field is smeared out by the finite size of the nucleus, the states corresponding to $j\neq 0$ will also exist. The interesting point is that the position of the energy levels for a smeared Coulomb field will not depend on the character of the smearing to the first order of perturbation theory. Thus, if one takes the equations of Corben and Schwinger and eliminates their quantity G (related to the 4th component of the 4-vector-Eq. (35)) by use of the subsidiary condition, then the first two equations (Eq. (41)) may be put in the form of simultaneous eigenvalue equations for F_1 and F_2. Linear combinations of these give the coupled equations for F^1 and F^2 (Eq. (36)). These equations have the form of Klein-Gordon operators on the left, and small perturbing terms on the right. These equations may now be solved by a perturbation method expanding in powers of $(Z\alpha)$.[2] The zero order terms are chosen to be the non-relativistic Schroedinger equations. The first order correction then enables one to compute the corrections to the scalar meson levels. No cut-off for the Coulomb field is necessary in this order. This would not be the case in higher orders.

The corrections to the scalar meson level for $n=1$, $j=1$, "l"$=0$ (ground state) was found to be $+2/3(Z\alpha)^4\mu c^2$ (μ is the mass of the meson). The state $n=2$, $j=1$, "l"$=0$ was found to be shifted by the amount $+1/12(Z\alpha)^4\mu c^2$, which makes this state and the state $n=2$, $j=2$, "l"$=1$ (which is unshifted) coincide at the scalar level $2p$. For $n=3$, the $j=1$, "l"$=0$ level again coincides with the "l"$=1$ level after correction. The $j=2$, "l"$=1$, the $j=3$ "l"$=2$, and the $j=1$ "l"$=2$ levels are unchanged from the corresponding scalar levels.

If this same treatment is applied to the radial equations for a meson of arbitrary magnetic moment $(1+\gamma)$, it is found that a cut-off would be necessary in the first order for states of "l"$=0$.

It is interesting to note that if the π-meson were a vector meson, the shift of its lowest energy level (compared to the scalar level) would be of the same order of magnitude as its broadening due to the effect of nuclear absorption,[2] except for the case of hydrogen where it may be larger by an order of magnitude.[3]

I am greatly indebted to Dr. R. E. Marshak and Dr. A. S. Wightman for many valuable discussions.

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[3] R. E. Marshak and A. S. Wightman, Phys. Rev. **76**, 114 (1949).

A Precise Method of Determining the Faraday by Magnetic Resonance

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A METHOD of determining the Faraday has been developed in which the cyclotron resonance frequency of protons in a known magnetic field is measured. The ions are formed by electron impact as in the usual mass spectrometer. They are prevented from escaping axially by means of a d.c. electric field in the same way that electrons are trapped in the Phillips ion gauge. A uniform r-f electric field of variable frequency at right angles to the magnetic field accelerates ions of a selected charge-to-mass ratio at resonance until they attain a radius of 1.0 cm at which point they strike the collector and the current is measured with an electrometer tube amplifier. An ion having a different charge-to-mass ratio cannot attain a radius of this magnitude unless the frequency is tuned for ions of this type. The width of the resonance peak depends on the number of cycles in the time required for the ions to reach the collector. This width may be decreased by decreasing the amplitude of the r-f voltage supplying the field and therefore the limiting resolution depends on the length of time it is possible to trap the ion and still detect resonance. The peak width is also limited by the uniformity of the magnetic field as in nuclear resonance.

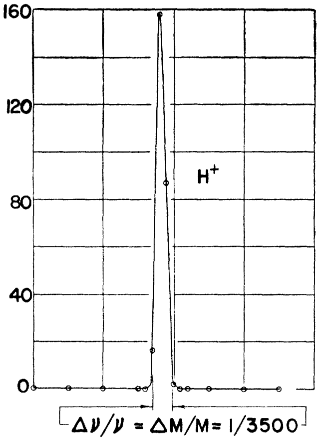

FIG. 1. Peak shape for protons as the frequency is varied and the magnetic field held constant at approximately 4700 gauss.

Figure 1 shows the peak shape for protons as the frequency is varied and the magnetic field held constant at approximately 4700 gauss (resonance frequency about 7 Mc). Higher resolution than this is attainable, but this was a convenient width for the initial experiments with the frequency control readily available. It was found that the instrument was so sensitive that the residual gas pressure could be used so no hydrogen was admitted to the tube. An ion gauge on the pumping arm near the tube indicated a pressure of 3×10^{-7} mm Hg. An electron current of 10 μa was used and the peak shown in Fig. 1 corresponds to an ion current of approximately 10^{-13} amp.

The magnet is the same one used in the measurement of the gyromagnetic ratio of the proton[1] and the resonance frequency was measured in relation to the nuclear resonance frequency of the proton. For this purpose the two resonance probes (cyclotron and nuclear) could be quickly interchanged while a third nuclear resonance probe held the field constant. We wish to emphasize the fact that the value reported in this note is a preliminary one until a more exhaustive study is made for possible systematic errors. This preliminary result is

$$\nu_c/\nu_n=0.358106\pm 0.000010,$$

where ν_c= cyclotron frequency of the proton, and ν_n= nuclear resonance frequency of the proton. The Faraday is obtained from the relation

$$F=\gamma M_p[\nu_c/\nu_n],$$

where γ= gyromagnetic ratio for proton $=(2.67524\pm 0.00020)\times 10^4$ (no diamagnetic correction) sec.$^{-1}$ gauss^{-1},[2] and M_p= isotopic weight of $H^+=1.007580\pm 0.000003$.[3,4]

The Faraday is then

$$F=9652.8\pm 0.8 \text{ e.m.u./g (physical scale).}$$

This is to be compared[5] with the value 9650.5 from the silver voltameter and 9652.2 from the iodine voltameter. (This is discussed in reference 4.)

The accuracy will be limited principally by the uncertainty in the gyromagnetic ratio. Work is in progress here which should considerably decrease the uncertainty in the gyromagnetic ratio. The agreement of our result with that of the voltameter gives added support to the value of e/m for the electron[6,7] resulting from our value of γ.

If our value of the ratio ν_c/ν_n is combined with the measurement of Gardner and Purcell,[7] one obtains a value of the ratio of the mass of the proton to the mass of the electron of greatly improved accuracy. This new result is

$$M_p/m_e = 1835.979 \pm 0.056.$$

Several possible applications of this instrument suggest themselves. In the first place, it looks very promising for the measurement of packing fractions. Its simplicity, high sensitivity, and variable resolution should make it useful in many other research and analytical applications. Since this device measures ω, it is suggested that it be called the omegatron.

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Theory of the Electric Resistivity of Polycrystalline Graphite*

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IT is possible to explain the observed temperature dependence of electric resistivity in commercial polycrystalline graphite by an extension of the theory of single crystal graphite as developed by Wallace.[1] There are two mechanisms required. First, one assumes that the conduction in the polycrystalline sample takes place in the planes of cleavage of the individual crystals. The large anisotropy (of the order of 10^4) prevents any appreciable conduction across the planes. This confinement of the current to the direction of orientation of the crystals, which will not necessarily be in the direction of gross current flow, increases the resistivity of the polycrystalline material over that of a single crystal in the plane of cleavage by a geometrical factor, s, depending on the ratio of the principal dimensions of the crystals, and their orientation with respect to gross current flow. For completely random orientation and a common ratio of crystal dimensions of 10, this factor is about 7. Second, it is assumed that the electron waves are scattered off the crystallite boundaries. Since these boundaries are regions of disorder, there is almost a certainty of scattering at these regions. Thus the probability of scattering per unit time can be estimated from the size of the crystallites, and the velocity of the waves in the crystal. Using estimates of crystallite size based on the width of x-ray diffraction lines, one computes the scattering probability per second to be 10^{14}. Furthermore, this probability should be temperature independent since the percentage change in crystallite dimension with temperature will be small. Adding this scattering probability to the thermal lattice scattering probability in Wallace's formula and multiplying by the geometrical factor s, one has

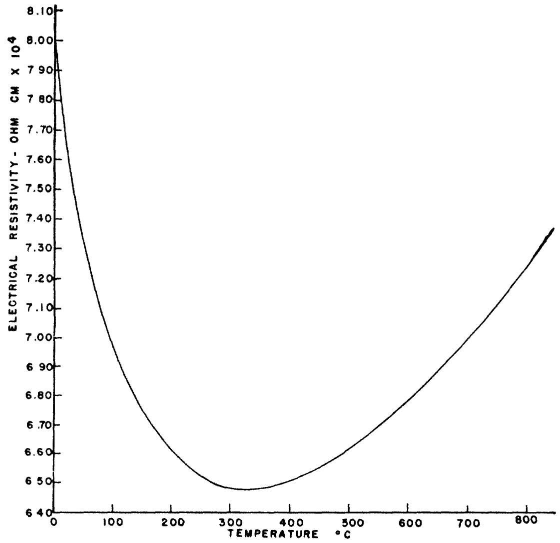

FIG. 1. Typical plot showing the variation of resistivity with temperature for polycrystalline graphite.

$$\rho = \frac{h^2 cs}{16\pi e^2 kT \ln^2}\left(\frac{1}{\tau_T} + \frac{1}{\tau_b}\right),$$

where the τ's are the inverse scattering probabilities.

By computing τ_T from the original Wallace equation, and measurements on single crystals, a typical plot showing the variation of resistivity with temperature for polycrystalline graphite has been reproduced in Fig. 1.

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A Note on the Variational Method for the Scattering Problem

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AS we have already pointed out,[1] Hulthén's formulation of the variational principle[2] has the advantage over other formulations[3] in that a more flexible trial wave function can be used. But in his practical applications he adopts a method which leads to a very inconvenient numerical procedure. Moreover, the manner of his application does not utilize the variational principle in its strict form: one of the equations he uses is not logically connected with the principle. In this note we shall indicate how we can improve Hulthén's method by making it more rigorous and at the same time much simpler for practical purposes. We shall illustrate the proposed method by considering S-scattering by a potential field of the Yukawa type.

Considering for the sake of simplicity only S-scattering, we define, following Hulthén

$$\mathfrak{L} = \int_0^\infty \psi(H-k^2)\psi dr, \tag{1}$$

where

$$H = -d^2/dr^2 + V(r). \tag{2}$$

It can then be shown that

$$\delta\mathfrak{L} = 2\int_0^\infty \delta\psi(H-k^2)\psi dr + k\delta\eta, \tag{3}$$

where η represents the phase shift. After obtaining this equation Hulthén suggested a variational method in which

$$\mathfrak{L} = 0 \tag{4}$$

is used to determine η while

$$\delta\mathfrak{L} = 0 \tag{5}$$

is used for obtaining a set of equations for the variational parameters. In this manner with a trial function of the form

$$\psi(r) = f(r)\cos\eta\,\sin kr + g(r)\sin\eta\,\cos kr, \tag{6}$$

Hulthén obtains a system of equations each of which is of the second degree in $\tan\eta$ and solves them by a method of successive approximations.